$$\Rightarrow \quad U_n - 1 \leq \int_1^n \frac{1}{\sqrt{x}}\, dx$$

$$\Rightarrow \quad U_n - 1 \leq \left[2\sqrt{x}\right]_1^n$$

$$U_n - 1 \leq 2\sqrt{n} - 2$$

$$\Rightarrow \quad \boxed{U_n \leq 2\sqrt{n} - 1}$$

par suite on

$\forall p \geqslant 1$ on

$$\int_p^{p+1} \frac{1}{\sqrt{x}}\, dx \leq \frac{1}{\sqrt{p}}$$

$$\Rightarrow \quad \sum_{p=1}^{n} \int_p^{p+1} \frac{1}{\sqrt{x}}\, dx \leq \underbrace{\sum_{p=1}^{n} \frac{1}{\sqrt{p}}}_{U_n}$$

$$\Rightarrow \quad \int_1^{n+1} \frac{1}{\sqrt{x}}\, dx \leq U_n$$